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NOTE

Optimum Catalyst Pellet Geometry

In the design of catalytic reactors the effect of intraparticle diffusion is customarily expressed by means of the so-called effectiveness factor E, which represents the ratio of the actual conversion rate per unit volume of pellet to that which would prevail in the absence of any diffusional retardation. In the isothermal case, theoretical derivations of E lead to the general expression (1)

$$E = \phi(h') \tag{1}$$

where h' is the so-called Thiele Modulus and a function of particle geometry, particle diffusivity, and the intrinsic reaction rate constant.

The form of ϕ will in general depend on the particle geometry, but it was shown by Wheeler (*2*) and later in more detail by Aris (*3*) that the results for all shapes will lie close together if the characteristic dimension of the particle is taken to be V_p/S_p, the ratio of its volume to its external area. This enables us to use the simplest available form of E, which is the case of a first order reaction in a flat plate (*1*):

$$E = (1/h') \tanh h' \tag{2}$$

where

$$h' = \left(\frac{V_p}{S_p}\right)\left(\frac{k \cdot \sigma}{D}\right)^{1/2} \tag{3}$$

k represents the intrinsic reaction rate constant per unit surface area, D the effective diffusivity, and σ the surface area per unit pellet volume.

It is evident that a reduction of particle size will diminish the diffusional effects and hence increase the effectiveness factor. Another way of achieving this is through the use of hollow particles, e.g. the Raschig ring type. The benefits to be derived from this particular geometry will evidently go through a maximum, since a progressive increase in the internal diameter of the hollow particle will entail a simultaneous decrease in the available catalyst material per unit reactor volume. The purpose of the present note is to investigate more fully the advantages of a hollow particle geometry and to compare the conversion rate with that of a bed of solid particles. The pellets to be considered here are assumed to be cylindrical, external radius R_o and length $L = 2\,R_o$. The bed void fraction of the solid particles ϵ_b is assumed constant at 0.30. For the purpose of this paper it is convenient to define a modified effectiveness factor

$$E' = \frac{\text{Actual rate per unit reactor volume}}{\text{Rate per unit pellet volume without diffusional effects}} \tag{4}$$

$$E' = (1 - \epsilon_b)E$$

Applying Eqs. (2) and (4) to pellets of cylindrical shape we obtain for solid cylinders:

$$E'_s = (0.7/h) \tanh h \tag{5}$$

Similarly, by substituting the proper values of V_p and S_p in Eq. (3) and combining with Eqs. (2) and (4) the result for hollow cylinders becomes:

$$E'_h = \frac{0.7(1 + R_i/R_o)(1 - \frac{1}{3}R_i/R_o)}{h} \tanh\left(\frac{1 - R_i/R_o}{1 - \frac{1}{3}R_i/R_o}\, h\right) \tag{6}$$

h represents the Thiele modulus for the solid cylinders. In Fig. 1, the ratio E'_h/E'_s has been plotted against h with R_i/R_o as parameter. A number of interesting facts

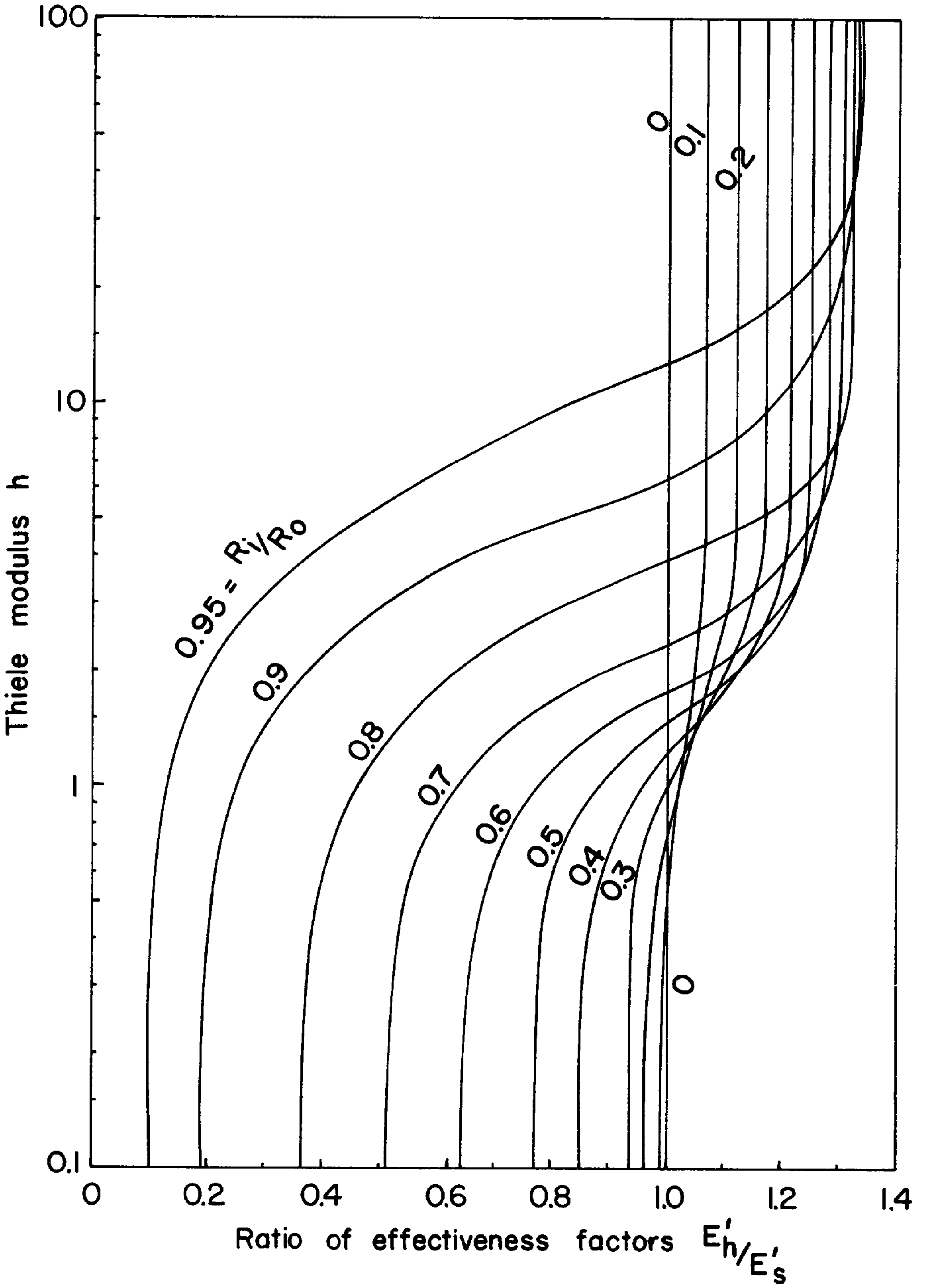

FIG. 1. Effectiveness factor ratio as a function of Thiele Modulus and particle radius ratio.

emerge from the plot. It is seen that below values of $h = 1.0$, the hollow particles are in general inferior to the corresponding solid shapes, irrespective of the ratio R_i/R_o. This is to be expected, since diffusional effects do not ordinarily become noticeable much below these values [see Fig. 2, ref.

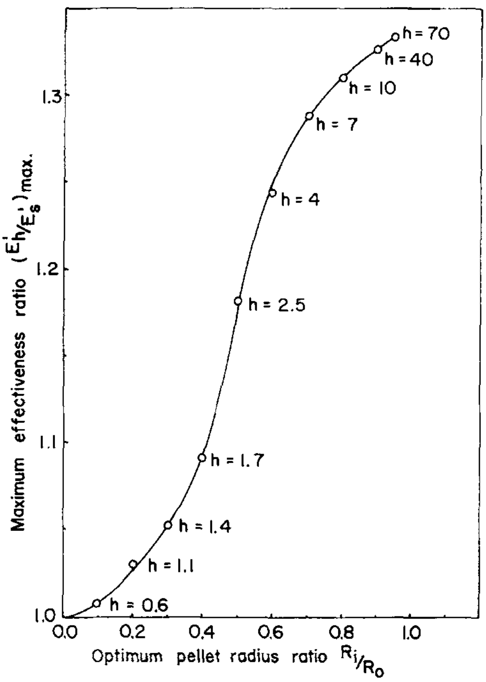

FIG. 2. Optimum radius ratio for hollow cylinders.

(*3*)]. As h and hence the diffusional resistance increases, the effectiveness ratio rises above 1 and tends towards the limiting value

$$(E'_h/E_s')_{lim} = (1 + R_i/R_o)(1 - \tfrac{1}{3}R_i/R_o) \quad (7)$$

The ratio reaches its maximum value of 1.33 as $R_i/R_o \to 1$; in other words, cylindrical pellets which are hollowed out to the extent of 80% or more of their volume will be roughly 30% more effective than their solid counterparts, provided the diffusional resistance is sufficiently high. While this does not represent a spectacular increase in reactor efficiency, it does hold out the prospect of considerable savings in catalyst material and/or a reduction in pressure drop along the reactor. These advantages have, of course, to be balanced against the reduced strength of the hollow pellets compared to the solid cylinders.

Optimum values of R_i/R_o corresponding to maximum effectiveness ratios were read off Fig. 1 and are presented in Fig. 2. The plot provides a means for rapidly determining optimum pellet geometries for a given value of h. Due to the sensitivity of the curve to small changes of the parameter, the method is only approximate, but more accurate values may be obtained, if so desired, by analytical optimization of Eq. 6. It may be noted that at values of h below the order of 1 the results given in both Figs. 1 and 2 apply quantitatively to higher order reactions as well as the nonisothermal cases treated in the literature (*4*), since the pertinent corrections do not involve the pellet geometry.

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